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STATICAL HYSTERESIS IN CYCLES OF EQUAL LOAD RANGE

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By G. H. Keulegan

ABSTRACT

The present paper deals with an investigation made for the purpose of ascertaining the influence of the extreme loads on the energy hysteresis loss in the cycles of equal load range during the flexure of an Armco iron bar. It is an extension of work reported in Bureau of Standards Technologic Paper No. 332. The flexure of the bar is considered for the case where one end of the bar is clamped and the other end loaded. Since in Armco iron the hereditary hysteresis is negligible in comparison with the statical hysteresis, the results of the experiment apply mainly to statical hysteresis.

Cycles of small extreme loads only are considered, and the conclusion derived from the result of the experiments is to the effect that the energy loss due to statical hysteresis in cycles of equal load range is independent of the extreme loads.

CONTENTS

I. INTRODUCTION

The phenomena of statical hysteresis in bodies undergoing elastic deformation has been discussed [1] previously by the author. One of the main deductions of the theoretical discussion then presented was in reference to E_0, the energy loss per unit volume, which occurs during a closed stress cycle; the loss being assumed to be due mainly to statical hysteresis; that is, that part of hysteresis which is independent of elastic after working. The relation referred to is

$$E_0 = \frac{1}{3}\beta\sigma_m^3 \quad . \tag{1}$$

where σ_m represents the stress range of the cycles or the algebraic difference of the extreme values of the stresses, σ_1 and σ_2, of the cycles; β is the modulus of statical hysteresis, a characteristic constant for the material under stress. Applicability of relation (1) is

[1] G. H. Keulegan, Statical Hysteresis in the Flexure of Bars, B. S. Tech. Paper No. 332; 1926.

supposed to be restricted, first, to bodies in cyclic state and, secondly, to stresses far within the limits of the so-called elastic proportionality.

Fundamentally, two distinct methods of experimentation will suggest themselves, in order to prove for a given material the exactness or the falsity of the relation involved in equation (1). One of these methods may be looked upon as being explicit and the other as implicit. In the explicit method all the points of the body in deformation experience the same axial stresses for a given external load during a closed load cycle. Consider, for example, a rectangular bar in compression. If the compression load varies from $L=0$ to $L=L_m$ and is brought back to zero value, the loss of energy H due to statical hysteresis, according to equation (1), will be given by

$$H=\frac{1}{12}\frac{l}{a^2b^2}\beta L^3_m \tag{2}$$

where l is the length of the bar in compression, $2a$ is the thickness, b is the width. In the expression for H, the powers of the cross-section dimensions and of the length are less than that of the load range.

In the implicit manner of experimentation, however, the load is so applied that all points of the body deformed do not experience the same stress for a given external load, but vary from point to point. In a form of body for which the stresses are easily evaluated the expression for H also contains the dimensions of the body, which, however, are of a higher power than that of the load range. For example, in a rectangular bar clamped at one end and loaded at the other end the expression for H is

$$H=\frac{9}{64}\frac{\beta}{b^2a^5}l^4L_m{}^3 \tag{3}$$

where, again, a is the half thickness of the bar, b the width, l the length, and L_m the load range in the load cycle.

Each of these methods has its advantages and disadvantages. In the explicit manner of experimentation one may be always sure of the finality of the deductions drawn from a small number of experiments; it is sufficient to experiment with a rod of a given length and a given cross section subjected to various load ranges. The disadvantages of such experiments consist in the fact that the deformations are never accurately determined. On the other hand, in experiments of the implicit type, deformations are measured with sufficient accuracy; but, in order to insure correct deductions from the data, one is compelled to consider bars of various thicknesses and of various lengths. Unfortunately, such extended experimentation is hardly desirable.

In the paper cited above, recourse was made to the implicit manner of testing, the basis of experimentation being the verification of formula (3). Four different lengths of rectangular bars of Armco iron were considered. In each cycle studied the lower value of the extreme loads was invariably $L_1=0$, while $L_2=L_m$ was some positive quantity. The data obtained were in good agreement with equation (3). This, of course, proves indirectly the truth of equation (1) for that particular material. Now, proof of the method could have been made more complete had experiments also been made with load cycles in which the load range remained the same, but the extreme loads made to vary between negative and positive values. Consider such cycles—(1), the extreme loads are $L_1=0$ and $L_2=L_m$; (2), the extreme loads are $L_1=-\frac{L_m}{2}$ and $L_2=+\frac{L_m}{2}$, and (3), the extreme loads are $L_1=-L_m$ and $L_2=0$. Are the energy losses of statical hysteresis alike in these cycles? The purpose of the present paper is to seek an answer to this question. If experiments show that the losses are alike, then some further proof has been secured in favor of the relation given in equation (1).

II. APPARATUS

It is necessary for the purpose of the present experiments to be able to apply at will negative and positive loads at the free end of the bar in a manner decidedly free from external friction. All loads causing an upward deflection of the bar will be designated as negative loads. The application of this end of applying frictionless negative and positive loads was achieved successfully by utilizing the buoyancy of a wooden ring of rectangular cross section when immersed in a mercury bath. The sketch of the apparatus used is given in Figure 1. The rectangular bar *A*, whose hysteretic behavior is to be studied, is clamped in the block *B* by means of a heavy screw *O*. Block *B* is firmly attached to the thick plate *C* resting on a table. *D* is the circular container of mercury. The buoyant effect of block *E* exerts an upward force at the bar at the point *F*. *H* is a pan which can be loaded so as to control the amount of the upward force on the bar due to the buoyant ring. The deflection of the bar at the point *G* is read by the micrometer *N*, which is firmly supported in space by the arm *M* and rod *L*. *L* and *B* are fixed to the same plate *B*, and are, therefore, free from deflection relative to each other. The contact between the micrometer and the rectangular collar at *G* is determined electrically. The description and characteristics of the micrometer are given in the paper previously cited.

The deflections read by the micrometer were made at a point slightly displaced from the point of application of the load. When

the micrometer read 0.0204 mm the corresponding deflection of the bar in flexure was 0.0195 mm. If the deflections had been taken at the point *F*, an error would have been introduced due to the rotation of the collar at *F* in loading and unloading the pan; the collar at *G* is free of extraneous external forces, and, hence, the capsule on *G* is fixed in relation to the neutral axis of the bar in flexure.

III. EXPERIMENTAL PROCEDURE

The total weight of the pan and the wooden ring arrangement was 1 pound. This was equivalent to a positive force of 1 pound,

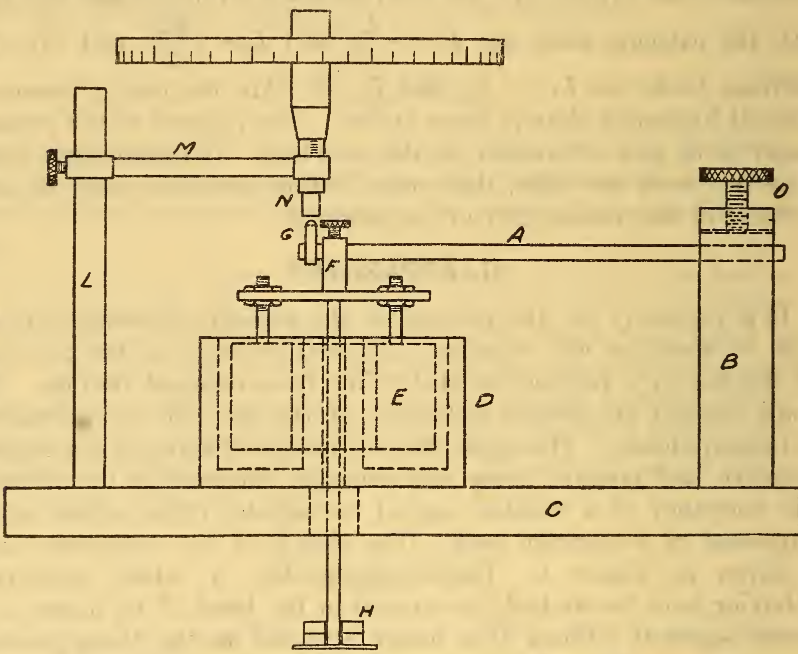

Fig. 1.—*Apparatus for producing and measuring the flexure*

which deflected the end of the bar downwards 0.2044 mm. By the gradual introduction of mercury and by the addition of small weights a definite end deflection of the bar could be secured which corresponded to a negative load of 7 pounds. This deflection was −1.6352 mm, which is eight times 0.2044 mm, the amount that the end deflects under a weight of 1 pound. Minus 7 pounds was the maximum negative load applied during the experiments. Any load which is greater, algebraically, than −7 pounds could be obtained, of course, by adding loads to the pan. The maximum positive load applied to the bar was 9 pounds.

Five cycles were considered with a load range L_m of 8 pounds or 3.63 kg; they are as follows:

Cycle No.	Extreme loads	
	Lower value, L_1	Upper value, L_2
	kg	*kg*
1	−3.18	+0.45
2	−2.27	+1.36
3	−1.36	+2.27
4	−.45	+3.18
5	+.45	+4.08

Prior to deflection measurements in a given cycle, the bar was put into the cyclic state for the particular load cycle. To achieve this, the particular cycle was repeated by applying alternately and quickly the extreme loads of the cycle. When the reading of the deflection corresponding to the lower value of the extreme loads assumes a constant value the bar was taken to be in the cyclic state for that particular cycle. To illustrate take cycle No. 3. Below are given the micrometer readings of the bar position for load $L_1 = -1.36$ kg. The first number refers to the position when the load $L = -1.36$ kg is applied for a long period, say, 20 hours or more; the second, third, etc., are the corresponding readings of the bar after successive repetitions of the load cycle, made in quick succession.

Micrometer reading in millimeters:

(1). 7.4828
(2). 7.4572
(3). 7.4544
(4). 7.4543
(5). 7.4542

Thus at the end of the fourth repetition in rapid succession of the cycles, the bar was adjudged to be in a cyclic state for that particular cycle.

In tests for the determination of the width of the hysteresis loop, the time required to complete a cycle varied from 25 to 30 minutes; that is, each increment of load was operative for a duration of 2 minutes, nearly. The air temperature during the tests ranged from 20° to 30° C., however, in each particular cycle the temperature did not change more than 0.5° C.

IV. ARMCO IRON SPECIMEN

The bar subjected to the tests described in the paper cited above was one of the prongs of a tuning fork of Armco iron. After having studied the damping characteristics of the oscillations of the fork, one of the prongs was separated from the stem. The bar thus

obtained also constitutes the specimen for the present tests. The chemical analysis of the material was made by the chemistry division of the Bureau of Standards. It contained, excluding iron, carbon, 0.016 per cent, and manganese, 0.03 per cent.

The width of the bar is 1.395 cm and the thickness $2a$ is 0.830 cm. The modulus of elasticity of the material, E, is 20.04×10^{11} dynes per square centimeter.

V. DISCUSSION OF DATA

The experimental data are given in Table 1. In this table appear the numerical designation of the cycles, the loads at which the deflection and width of hysteresis loops are measured, the width of the hysteresis loop in each of four runs, the average width of hysteresis loop, and the end deflection. It is to be remembered that these values of measured width of the hysteresis loop and the deflection are not the actual values of the deflection and width of hysteresis loop of the bar, but are 1.045 times greater, as was explained in the description of the apparatus. The average widths of the hysteresis loop as given in this table are plotted against load in Figure 2. The areas under these curves are indicative of the energy loss in each corresponding cycle. As the after effect or the residual displacement at the lower values of the extreme loads is a small quantity in comparison with the maximum width of the hysteresis loop, the loss of energy will be interpreted as due, mainly, to statical hysteresis.[2] To obtain the energy loss in each cycle the areas of the curves are evaluated in apparent ergs and reduced to ergs by dividing by 1.045, due to the fact that the true deflection of the bar is that much smaller. The loss in each cycle is tabulated in Table 2.

TABLE 1.—*End deflection and width of the hysteresis loop in load cycles of the same range but of different extreme loads*

Cycle No.	Load	Width of hysteresis loop				Average	End deflection
		Run 1	Run 2	Run 3	Run 4		
	kg	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*
1	−3.175	0.0001	0.0002	+0.0000	0.0000	0.0001	0.000
	−2.722	.0034	.0038	.0031	.0029	.0033	.205
	−2.268	.0062	.0059	.0055	.0053	.0057	.412
	−1.814	.0072	.0073	.0069	.0064	.0069	.618
	−1.361	.0082	.0080	.0079	.0074	.0079	.826
	−.907	.0076	.0071	.0070	.0066	.0072	1.032
	−.454	.0045	.0058	.0057	.0054	.0053	1.230
	.000	.0043	.0025	.0032	.0030	.0032	1.445
	+.454	.0000	.0000	.0000	.0000	.0000	1.657
2	−2.268	.0011	.0006	.0001	.0003	.0005	.000
	−1.814	.0044	.0040	.0033	.0042	.0040	.204
	−1.361	.0060	.0060	.0055	.0047	.0056	.411
	−.907	.0073	.0072	.0070	.0067	.0070	.616
	−.454	.0080	.0071	.0076	.0075	.0076	.823
	.000	.0067	.0065	.0065	.0066	.0066	1.031
	.454	.0051	.0052	.0051	.0055	.0052	1.238
	.907	.0031	.0031	.0034	.0033	.0032	1.446
	1.361	.0000	.0000	.0000	.0000	.0000	1.650

[2] The justification of this statement will be discussed in a paper to appear under the title "Hereditary hysteresis in some metals."

TABLE 1.—*End deflection and width of the hysteresis loop in load cycles of the same range but of different extreme loads*—Continued

Cycle No.	Load	Width of hysteresis loop				Average	End deflection
		Run 1	Run 2	Run 3	Run 4		
	kg	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*
3	−1.361	0.0011	0.0009	0.0006	0.0000	0.0004	0.000
	−.907	.0040	.0038	.0043	.0036	.0039	.204
	−.454	.0065	.0067	.0076	.0068	.0069	.410
	.000	.0072	.0082	.0081	.0080	.0079	.616
	.454	.0084	.0087	.0079	.0075	.0081	.823
	.907	.0077	.0074	.0074	.0073	.0075	1.030
	1.361	.0050	.0057	.0057	.0057	.0055	1.231
	1.814	.0029	.0034	.0040	.0031	.0034	1.443
	2.268	.0000	.0000	.0000	.0000	.0000	1.648
4	−.454	.0012	.0009	.0003	.0000	.0006	.000
	.000	.0042	.0033	.0038	.0034	.0036	.205
	.454	.0062	.0056	.0061	.0048	.0057	.410
	.907	.0074	.0071	.0069	.0061	.0069	.616
	1.361	.0088	.0078	.0078	.0079	.0081	.820
	1.814	.0075	.0070	.0075	.0064	.0071	1.026
	2.268	.0050	.0040	.0053	.0050	.0058	1.232
	2.722	.0035	.0028	.0030	.0030	.0031	1.437
	3.175	.0000	.0000	.0000	.0000	.0000	1.642
5	.454	.0010	.0004	.0006	.0004	.0006	.000
	.907	.0040	.0036	.0037	.0031	.0036	.205
	1.361	.0059	.0064	.0059	.0060	.0060	.409
	1.814	.0072	.0066	.0068	.0056	.0065	.614
	2.268	.0077	.0070	.0080	.0070	.0074	.817
	2.722	.0069	.0064	.0065	.0064	.0065	1.022
	3,175	.0054	.0054	.0056	.0045	.0053	1.227
	3.629	.0031	.0027	.0028	.0029	.0029	1.431
	4.082	.0000	.0000	.0000	.0000	.0000	1.635

TABLE 2.—*Energy loss H due to statical hysteresis*

Cycle No.	Extreme loads in kg		*H* ergs
	Upper value	Lower value	
1	−3.17	+0.45	1,670
2	−2.27	+1.36	1,660
3	−1.36	+2.27	1,855
4	−.45	+3.17	1,720
5	+.45	+4.08	1,645
Average			1,715

Thus it appears that the energy losses H in the individual cycles do not differ much from each other. The average loss in these cycles is 1,715 ergs; the value of H in cycle No. 3 has the largest deviation from this mean, the deviation being 9 per cent of the average value of H. However, considering the errors of measurement this deviation is in the neighborhood of the experimental error, and, therefore, one is not much in error in stating that the energy losses of statical hysteresis in cylces of equivalent load range are equal when the maximum fiber stresses are well within the so-called proportional limit. Such a conclusion, without doubt, is in conformity with the relation involved in equation (1).

VI. A REDETERMINATION OF THE HYSTERESIS MODULUS OF ARMCO IRON

It is an interesting point that the hysteresis modulus calculated from the data of these experiments agrees with the one previously determined. According to equation (3)

$$\beta = \frac{64}{9} \frac{b^2 a^5 H}{l^4 L_m^3} \tag{3a}$$

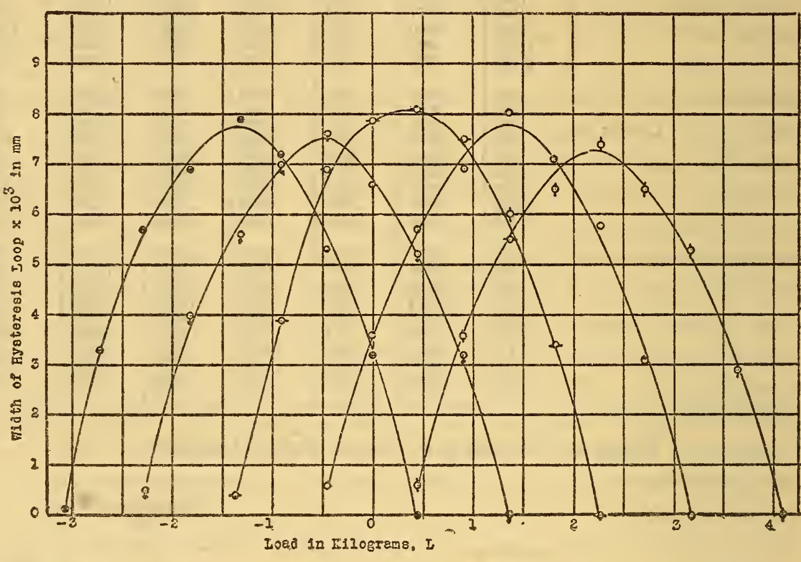

FIG. 2.—*Width of hysteresis loop in cycles of load range of 3.63 kg*

To evaluate β, the value of l, the equivalent length of the bar in deflection, must be determined experimentally. To obtain this quantity the following formula was used [3]

$$l^3 = 2ba^3Ed \tag{4}$$

where d is deflection per unit load. In these experiments $d = 0.430$ mm per kilogram, $E = 2.05 \times 10^6$ kg per square centimeter, $a = 0.415$ cm, and $b = 1.395$ cm, therefore, $l = 25.99$ cm. Substituting the value of $H = 1{,}715$ ergs, $l = 25.99$ cm and $L_m = 3.63$ kg in equation (3a), there results

$$\beta = 1.33 \times 10^{-5} \text{ ergs cm}^3 \text{ per kg}^3$$

This value agrees exactly with the one previously given.

[3] See footnote 1, p. 379.

VII. SUMMARY

In the flexure of a bar of Armco iron, one end fixed and the other end loaded, the energy loss due to statical hysteresis is constant in closed load cycles of equivalent load range, provided that the values of the extreme loads are small in these cycles. The value of the hysteresis modulus of Armco iron was redetermined and found in agreement with the results of previous experiments.

WASHINGTON, September 20, 1927.